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ABSTRACT

This teacher's guide contains instructions for the use of a booklet designed to teach problem solving through guessing. For each lesson the purpose, an overview, and detailed teaching procedures are presented. The five lessons are: (1) using guesses and computation to solve problems; (2) using guesses to solve two-step problems; (3) using guesses and diagrams to solve problems; (4) using estimation to solve measurement problems; and (5) using guesses to solve problems. (MP)

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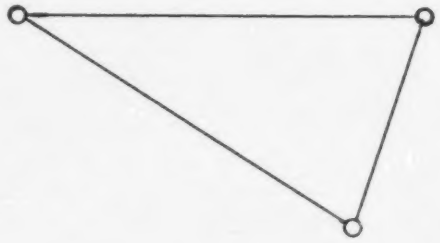

Indiana University
Bloomington

Mathematical Problem Solving Project

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INTRODUCTION TO PROBLEM SOLVING

One of the principal goals of teaching mathematics is developing problem solving skills for children. It is a complex task and a real challenge for every teacher. It is easier to analyze this task if we identify four stages that most successful problem solvers go through. These stages are:

STAGE 1 Getting to know the problem

STAGE 2 Deciding what to do

STAGE 3 Doing it

STAGE 4 Looking back

We can better understand how to teach problem solving skills by analyzing the essential elements at each stage.

STAGE 1 GETTING TO KNOW THE PROBLEM

To solve a problem, the student not only must understand the words, but must also recognize the relation between the crucial elements. For example, a student may be able to read a problem about a spring, but unless he also recognizes that a spring stretches uniformly as weights are added, he will not be able to solve the problem.

The problem solver not only needs to identify the pertinent facts, but must also clearly identify the question that is being asked. Many students fail to get the correct answer because they answered the wrong question.

STAGE 2 DECIDING WHAT TO DO

The skills necessary at the "deciding what to do" stage are very different from those at the "get to know the problem" stage. To be a successful problem solver, a student needs a variety of tools in addition to computation skills. These tools should include the ability to make good guesses, make tables, make

diagrams, make graphs, write equations, and use other resources. The problem solver must choose from the tools he has available and make a plan to solve the problem. The modules in the Problem Solving Project are designed to teach the necessary tools and provide experiences in problem-solving situations.

STAGE 3 DO IT

This "do it" stage is closely related to Stage 2, "deciding what to do." A problem solver seldom chooses an attack that he doesn't know how to do. Therefore, this stage requires mastery of the tools discusses in Stage 2. The teaching of computational procedures alone is not enough to develop good problem solving skills. The invention of the hand calculator puts accurate computation within the grasp of all students, but it does not help them make the decisions that successfully solve problems.

STAGE 4 LOOK BACK

This stage is often ignored because once a student has an answer, it is on to the next problem. To get the most out of the effort, we first need to be sure that the answer is correct. Then we need to take time to learn as much as possible from the experience of solving the problem. Just suggesting that the student "check the answer" and make sure the answer is "properly labeled" is not enough. Some teachers have increased the number of correct answers by using a "check" sheet that a student completes each time he solves a problem. The elements in this check sheet vary, but they include: Can you rephrase the problem with your answer? Does your answer fit your original estimate for the answer? Have you labeled your answer? This "check sheet can be written on the board or made available in a duplicated form. It helps build good problem solving habits that increase the number of correct answers.

After the student has arrived at the correct answer, there is still a great deal to be learned by extending the solution. "Can you find an easier way to solve the problem?" or "How would you explain the solution to another student?" helps the student remember the processes the next time he needs to solve a similar problem. "Can you write another problem?" or "Can you think of a way to extend the problem to make it more interest-int?" can encourage the student to look for more general methods of problem solving. When we encourage the student to reflect on the solution, the student is more likely to recall the solution the next time a problem like the one he solved occurs.

Each of the four stages requires a different kind of learning activity. "Getting to know" the problem and "looking back" involve learning that must take place over time. They can't be taught in one day. The teachers should focus on these stages each time a student faces a problem. The "deciding what to do" and the "do it" stages are developed by concentrating on specific skills. The problem-solving modules in the Problem-Solving Project are designed to create this focus.

USING GUESSES TO SOLVE PROBLEMS

Many of the ways that we solve problems involve guesses and estimation. When you see an item a store marked one-third off, you usually don't compute, but you use your skill to estimate the cost before you decide whether you want the item. When you decide to fill up your gas tank, you estimate the cost to be sure you have enough money to pay the bill. Many problems can only be solved by making a guess and then checking your guess and sometimes you then make a better guess and continue until you have a solution.

When you teach the guessing strategy, you often have to overcome a negative attitude. This attitude is associated with computation. Of course you don't want students to substitute a guess to find the sum of a group of numbers. For this reason, many students are taught in the early grades that they shouldn't guess. There are different kinds of guesses. A random guess based on little or no information is usually of little value, but in most problem situations the student is making a guess upon the information that is given in the problem. The making of an educated guess is a skill that is necessary for problem solving. You wouldn't want the checkout clerk to guess at the cost of your groceries, but you make estimates when you choose the better buy at the grocery store. You may also estimate the total cost or to determine the largest head of lettuce for the money. The world around you is full of situations where estimation (making educated guesses) is a necessary and useful skill.

THE USING GUESSES TO SOLVE PROBLEMS MODULE

The purpose of the skills booklet is to teach basic entry skills so that the students can successfully solve problems at the lower levels in the problem solving deck. Most problem

solving skills are only introduced by teaching activities. These skills are then developed and extended by solving problems where the skills are necessary. Problem solving skills are not taught in one day or even in a series of days. They are developed and extended through on-going experiences over a long period of time. However, there must be a time when the skill is originally introduced or the student may never realize that the strategy exists. This is particularly true for the guessing strategy.

This skills booklet first introduces the idea that guesses are a desirable strategy in solving computation problems. Skills are then developed to solve two-step problems in Lesson 2. In Lesson 3 the student learns to use guesses in situations that require diagrams. The fourth lesson uses estimation skills in measurement problems. Lesson 5 uses guesses in general problem solving situations. After teaching the skills booklet, the problem deck provides for further use of the skills with problems involving all four skills at five levels of difficulty.

TO: THE TEACHERS

FROM: THE AUTHORS

We wrote this booklet in five lessons to be taught in five days. The lessons are:

Lesson 1 Using Guesses and Computation to Solve Problems . . . . . Pages 1-4

Lesson 2 Using Guesses to Solve Two-Step Problems . . . . . Pages 5-8

Lesson 3 Using Guesses and Diagrams to Solve Problems . . . . . Pages 9-12

Lesson 4 Using Estimation to Solve Measurement Problems . . . . . Pages 13-17

Lesson 5 Using Guesses to Solve Problems . . . . . Pages 18-24

We hope you will try grouping students in teams of four for the lessons. If you do use a team approach, you'll find that students with similar mathematical problem solving skills work more effectively together than do students with widely varied skills.

We wrote the booklet to be consumed. Each student is given a booklet and will use that booklet throughout the five-day sequence.

The lessons encourage the students to make guesses or estimates. Guessing is an essential first step in solving many problems and we encourage guessing throughout the module. The lessons also encourage the students to use information (perhaps by checking the first guess) to make a better guess or estimate. Finally, the student applies the technique to solving problems.

LESSON 1 USING GUESSES AND COMPUTATION TO SOLVE PROBLEMS

Purpose: Given a problem, the student will use guesses and computation to solve it.

Overview: In Lesson 1, we want the students to solve problems in which they compute until a specified result is obtained. We intend for the activities to increase their willingness to make a first guess. Computing with the initial guess gives information which helps the students make better second guesses. It may be necessary to repeat the guess-and-check cycle several times before the solution is found.

Teaching Procedure:

Pages 1 and 2 are teacher-directed.

Pages 3 and 4 may be worked by students at their own pace.

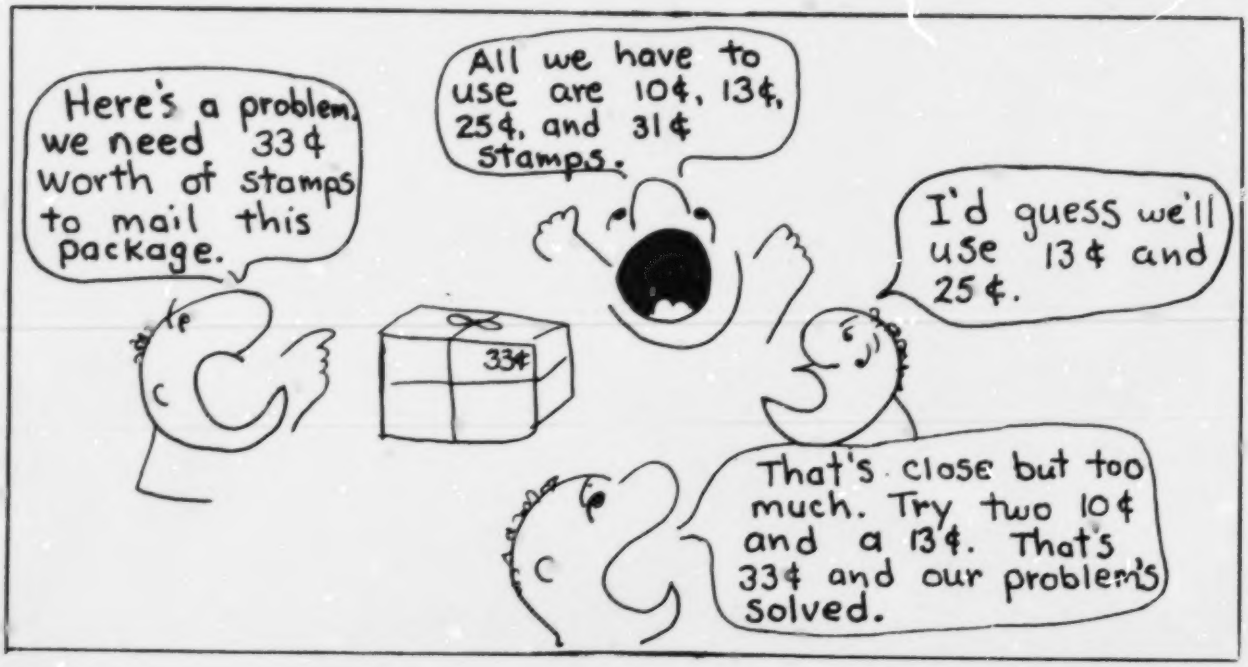
Pages 1 A and 1 B in the teacher's edition have problems which may be given to students who finish early.

Authors' Comments for page 1:

Read the cartoon and discuss the setting. Have students work in groups to solve problem 1. Discuss problem 1. Repeat the process for problems 2, 3, and 4.

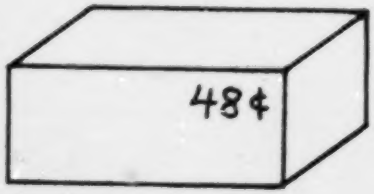
Your Comments for page 1:

LESSON 1: USING GUESSES AND COMPUTATION TO SOLVE PROBLEMS


Here's a problem, we need 33¢ worth of stamps to mail this package.
All we have to use are 10¢, 13¢, 25¢, and 31¢ stamps.
I'd guess we'll use 13¢ and 25¢.
33¢
That's close but too much. Try two 10¢ and a 13¢. That's 33¢ and our problem's solved.

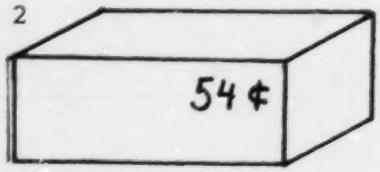
Guess which three stamps you use to mail these packages. Use only 10¢, 13¢, 25¢, and 31¢ stamps.

Problem 1.



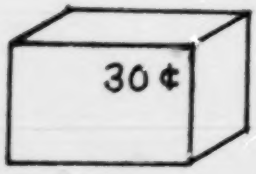
____ ____ ____

Problem 2



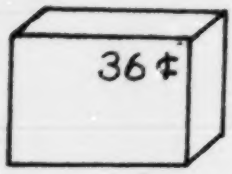
____ ____ ____

Problem 3.



____ ____ ____

Problem 4.



____ ____ ____

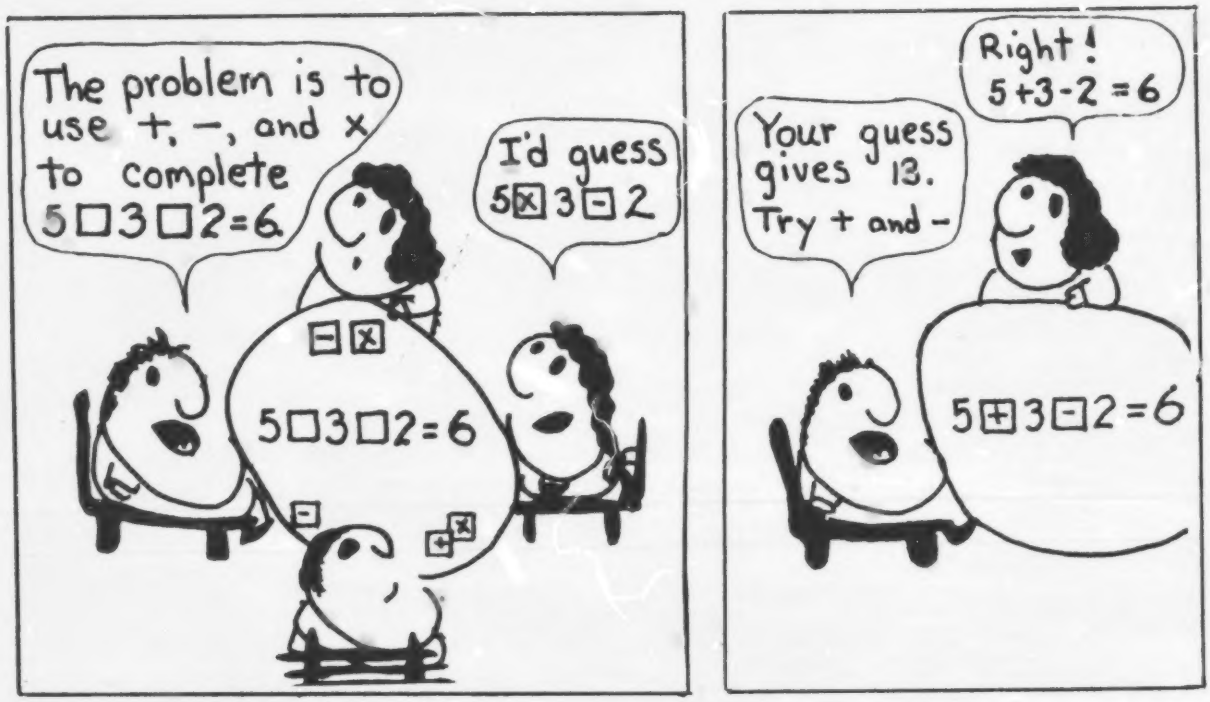
LESSON 1

Page 2

Authors' Comments for page 2:

Read the cartoon and discuss the setting. Have the students do problem 1. Discuss why some guesses are not correct. Repeat for problem 2. Students can guess-and-check for problems 3-6. When most of the students are finished, discuss the processes and solutions. Emphasize the need to check the guesses.

Your Comments for page 2:



Problem 1.

6 ☐ 4 ☐ 1 = 23

Ring the correct guess:

6 ⊠ 4 ⊞ 1 = 23

6 ⊠ 4 ⊟ 1 = 23

Problem 2.

6 ☐ 3 ☐ 4 = 7

Ring the correct guess:

6 ⊞ 3 ⊟ 4 = 7

6 ⊟ 3 ⊞ 4 = 7

USE +, −, AND x TO SOLVE THESE PROBLEMS.

Problem 3. 6 ☐ 2 ☐ 3 = 9

Problem 4. 4 ☐ 5 ☐ 8 = 28

Problem 5. 3 ☐ 6 ☐ 4 = 5

Problem 6. 2 ☐ 5 ☐ 4 = 14

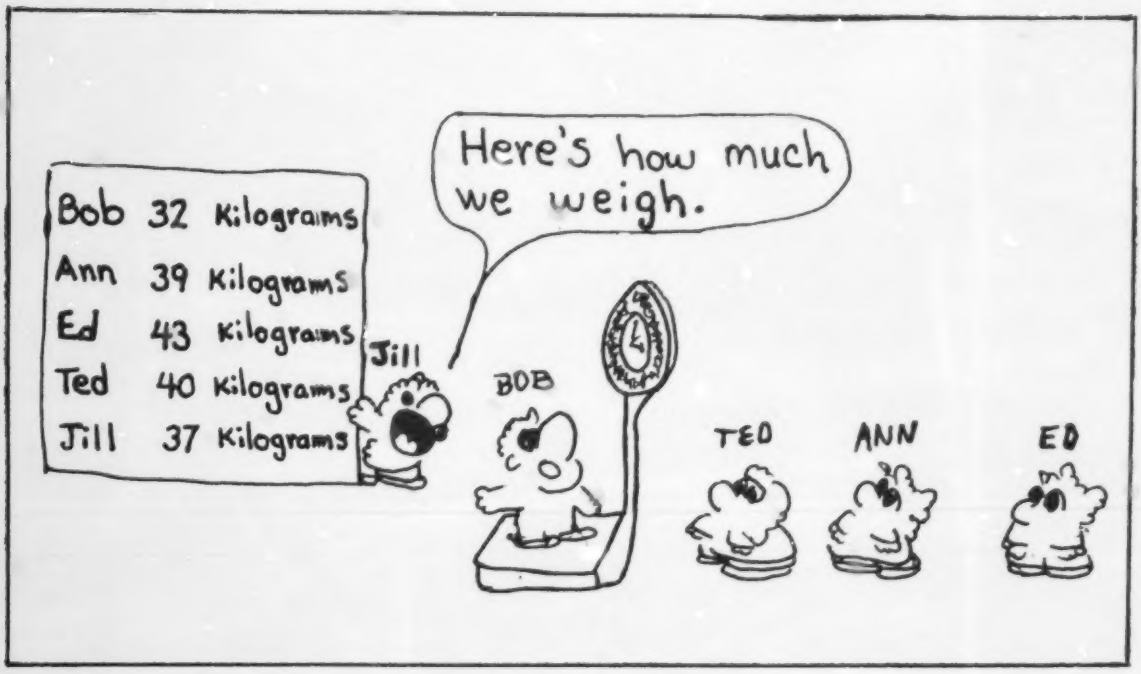
LESSON 1

Page 3

Authors' Comments for page 3:

Students may work in teams on page 3. Less teacher direction is needed for this page. We suggest that the students discuss their answers with each other before going on to the next page.

Your Comments for page 3:



USE GUESSING TO SOLVE THESE PROBLEMS.

1. Two boys together weigh 75 kilograms. Who are they?

   _____ _____

2. Ann and a friend weigh 76 kilograms together. Who is her friend? _____

3. Which two kids together weigh 79 kilograms?

   _____ _____

4. Which three kids together weigh 122 kilograms?

   _____ _____ _____

5. Which three kids together weigh 109 kilograms?

   _____ _____ _____

LESSON 1

Page 4

Authors' Comments for page 4:

Students may work at their own pace on this page. Again we encourage them to discuss their answers with each other.

Your Comments for page 4:

SOLVE THESE PROBLEMS BY MAKING GUESSES AND CHECKING THEM.

1. These items are for sale. What did you buy that costs this much?

52 cents Book and Pencils

68 cents _____

76 cents _____

92 cents _____

2. Use +, x, and - on the tabs to name 10, 24, 65, and 79.

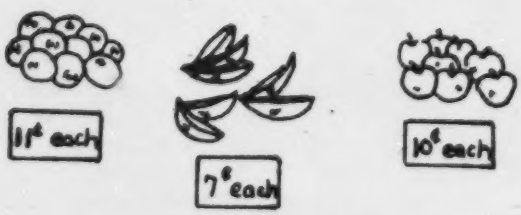
9 ◯ 8 ◯ 7 = 10

9 ◯ 8 ◯ 7 = 24

9 ◯ 8 ◯ 7 = 65

9 ◯ 8 ◯ 7 = 79

3. Bob bought apples, bananas, and oranges. He spent 46¢. How many of each did he buy? _____



LESSON 1

Pages 1 A and 1 B

OPTIONAL ACTIVITIES

Authors' Comments for pages 1 A and 1 B:

In our try-out experiences, we've found that lessons written for several age levels may be too long for some levels and too short for other levels. We have prepared these lessons so all students can complete the basic lesson in one period. We have provided optional activities for the early finishing groups. The optional activities for this lesson are labeled Lesson 1 A and Lesson 1 B. There are two copies of each of these pages. On each page are several problems. The problems on page 1 A (printed on blue) should be given to average and below average students. The problems on page 1 B (printed on yellow) are a little more difficult and should be given to above average students.

1. Cut the problems apart on the dotted lines. You may paste the problems on cards if you wish.
2. Early finishers get one problem at a time and solve it on a separate piece of paper.
3. As teams complete one of these problems, they may be given another. After one team finishes a problem, another team can use the same problem.

Your Comments for pages 1 A and 1 B:

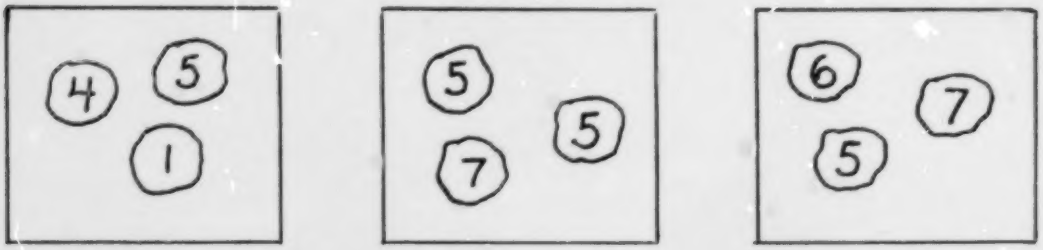
_____

_____

_____

_____

1. FOR EARLY FINISHERS.

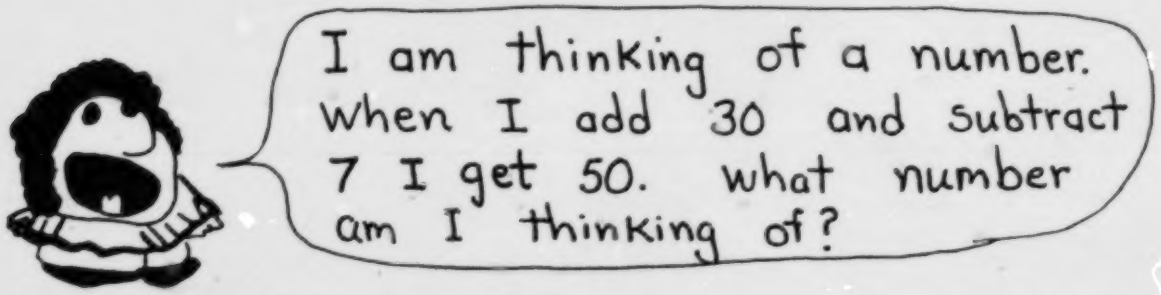


The tags can be moved so they add to 15 in each square. Show the tags in each square.

Lesson 1 A

---

2. FOR EARLY FINISHERS.



Lesson 1 A

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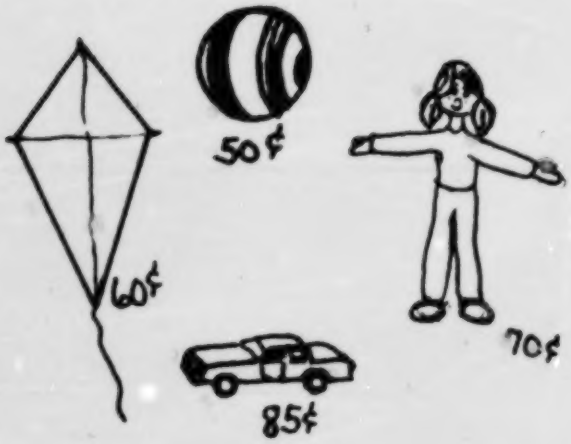
3. FOR EARLY FINISHERS.



I spent $1.95. What did I buy?

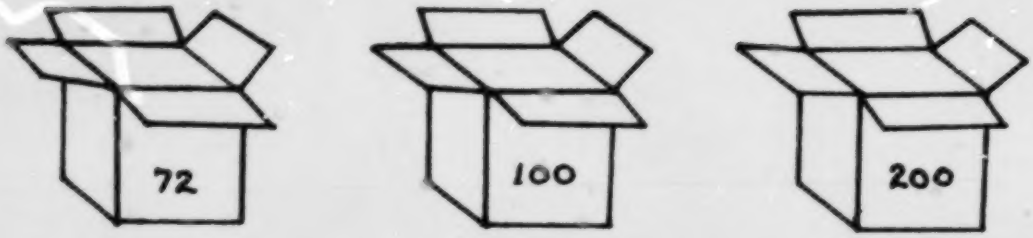
_____

_____

_____

Lesson 1 A

1. FOR EARLY FINISHERS.



There are 72 buttons in one box, 100 buttons in another box, and 200 buttons in the other. If you moved some of the buttons so that each box had the same number, how many would be in each box?

Lesson 1 B

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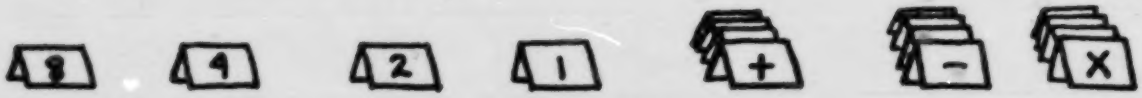
2. FOR EARLY FINISHERS.



Use the four numbers 1, 2, 4, and 8 and +, -, and x to name as many numbers as you can from 1 to 9.

Example: 8-4-2-1=1

LESSON 2 USING GUESSES TO SOLVE TWO-STEP PROBLEMS

Purpose: Given a problem involving two conditions, two variables, or two operations, the student will use guesses to solve it.

Overview: In Lesson 2 we want students to solve problems involving two steps. In two variable problems, the student will select a first guess by satisfying one condition and check the guess by seeing if it satisfies the second condition as well. Making a guess, even though it proves to be incorrect, is an essential first step since it may lead to a better second guess and eventually the solution.

Teaching Procedure:

Pages 5 and 6 are teacher-directed.

Pages 7 and 8 may be worked by students at their own pace.

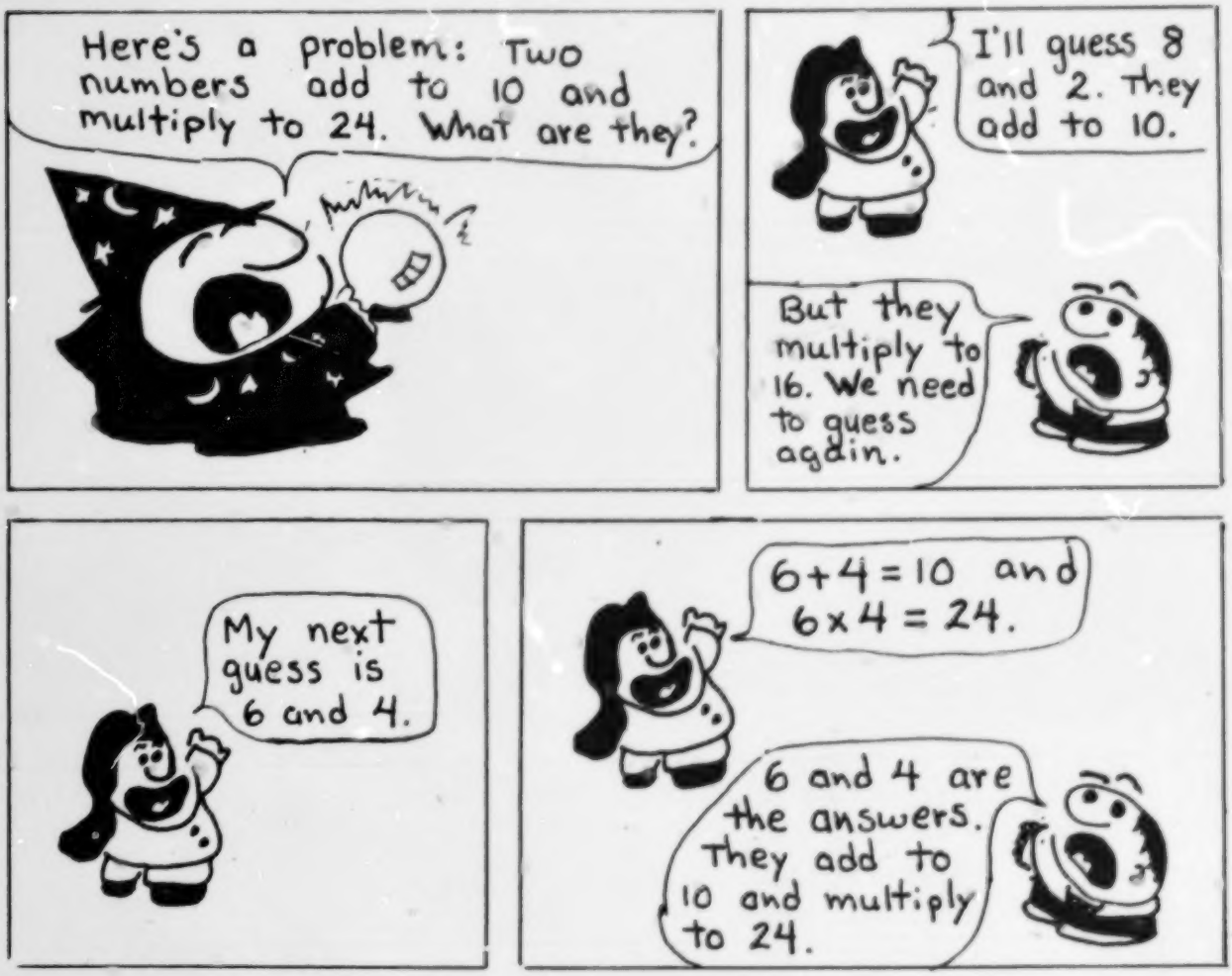
Pages 2 A and 2 B in the teacher's edition have problems which may be given to students who finish early.

Authors' Comments for page 5:

Read the cartoon. Discuss the problem and how the cartoon characters are solving it. Discuss problem 1 at the bottom of the page. Then repeat the process for problems 2, 3, and 4.

Your Comments for page 5:

LESSON 2: USING GUESSES TO SOLVE TWO STEP PROBLEMS.


Here's a problem: Two numbers add to 10 and multiply to 24. What are they?
I'll guess 8 and 2. They add to 10.
But they multiply to 16. We need to guess again.
My next guess is 6 and 4.
6+4=10 and 6x4=24.
6 and 4 are the answers. They add to 10 and multiply to 24.

SOLVE THESE PROBLEMS. GUESS THEN CHECK.

1. What two numbers have a sum of 20 and a difference of 4?

   _____ _____

2. What two numbers have a sum of 9 and a product of 20?

   _____ _____

3. When you add Fred's age and Monica's age you get 21. When you subtract them you get 3. How old are they? _____ _____

4. There are 24 students in a class. There are 2 more boys than girls. How many boys are in the class? _____

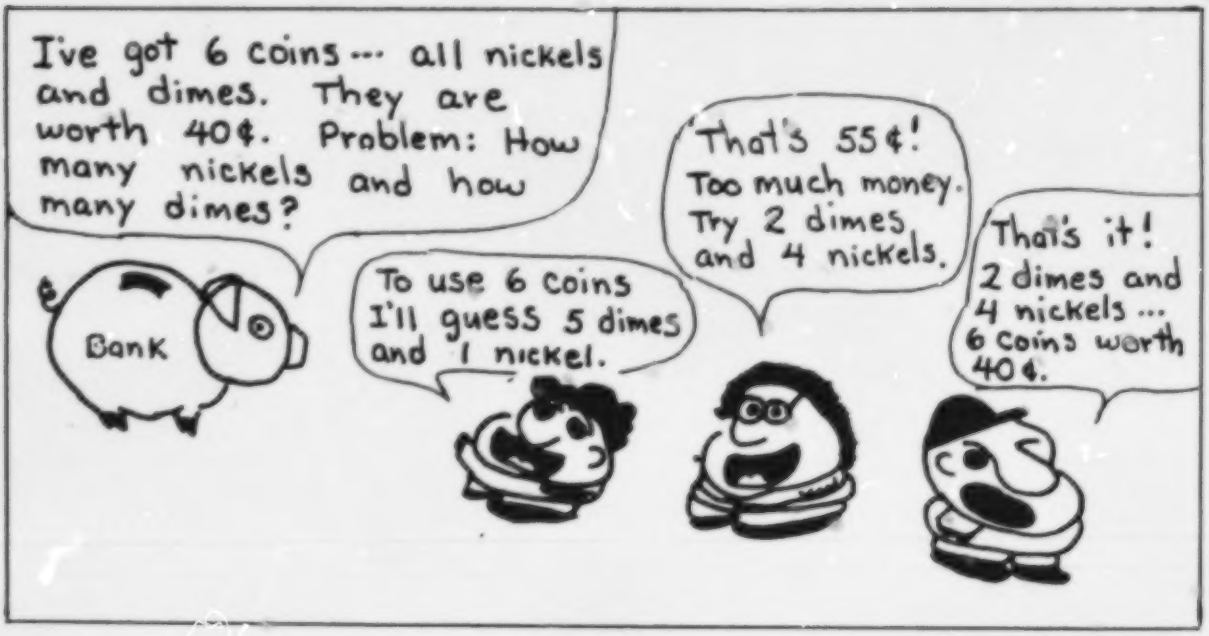
LESSON 2

Page 6

Authors' Comments for page 6:

Read the cartoon. Discuss the problem and how the cartoon characters solved it. Have the students do problem 1. Discuss what guesses are good if there are six coins (4 nickels and 2 dimes is a good guess even though it is not the correct solution. 1 nickel and 3 dimes is not a good guess because it does not use 6 coins.) Repeat for problem 2. Students can solve problems 3 and 4 individually or in teams. When most of the students are finished, discuss the processes and solutions.

Your Comments for page 6:


I've got 6 coins --- all nickels and dimes. They are worth 40¢. Problem: How many nickels and how many dimes?
Bank
To use 6 coins I'll guess 5 dimes and 1 nickel.
That's 55¢! Too much money. Try 2 dimes and 4 nickels.
That's it! 2 dimes and 4 nickels ... 6 coins worth 40¢.

USE GUESSING TO SOLVE THESE.

Problem 1. Six coins in a bank--all nickels and dimes. They are worth 35¢. What are they?

_____ nickels _____ dimes

Problem 2. Eight coins in a bank--all nickels and dimes. They are worth 65¢. What are they?

_____ nickels _____ dimes

Problem 3. Six coins in a bank--all dimes and quarters. They are worth $1.20. What are they?

_____ dimes _____ quarters

Problem 4. Eight coins in a bank--all dimes and quarters. They are worth $1.25. What are they?

_____ dimes _____ quarters

Authors' Comments for page 7:

Students may work in teams on page 7. Little teacher direction is needed for this page. We suggest that the students discuss their answers with each other before going on to the next page.

Your Comments for page 7:

Solve this problem. I'm thinking of just one number. when I double it, then add 10, I get 26. what is the number?
I'll guess 5.
Now I double 5 and add 10. I get 20.
2 x 5 + 10 = 20
That's too small.
I'm guessing 10.
That's bigger.
2 x 10 + 10 = 30
That's too big!
How about 8?
2 x 8 + 10 = 26
That's it! when you double 8 and add 10, you get 26!

SOLVE THESE PROBLEMS. GUESS THEN CHECK.

1. Bill has a lucky number. If you multiply it by 2 and subtract 4, you'll get 10. What is Bill's lucky number? _____

2. Sammy had some money. He spe 30¢, then earned 75¢. Now he has 95¢. How much did he start with? _____

3. A bus can seat 30 passengers. There is one empty seat for every two seats that are filled. How many passengers are on the bus? _____

LESSON 2

Page 8

Authors' Comments for page 8:

Students may work at their own pace on this page. Again we encourage them to discuss their answers with each other.

Your Comments for page 8:

SOLVE THESE PROBLEMS.

1. Joy had some money. She spent half of it and then got 10¢ more.

   Now she has 35¢.

   How much did she start with? _____

2. There are 28 students in a room.

   There are 6 more girls than boys.

   How many girls are in the room? _____

   How many boys? _____

3. The sum of Hank's age and Izaac's age is 19.

   The product is 90.

   Hank is older than Izaac.

   How old is Hank? _____

LESSON 2

Pages 2 A and 2 B

OPTIONAL ACTIVITIES

Authors' Comments for pages 2 A and 2 B:

In our try-out experiences, we've found that lessons written for several age levels may be too long for some levels and too short for other levels. We have prepared these lessons so all students can complete the basic lesson in one period. We have provided optional activities for the early finishing groups. The optional activities for this lesson are labeled Lesson 2 A and Lesson 2 B. There are two copies of each of these pages. On each page are several problems. The problems on page 2 A (printed on blue) should be given to average and below average students. The problems on page 2 B (printed on yellow) are a little more difficult and should be given to above average students.

1. Cut the problems apart on the dotted lines.
2. Early finishers get one problem at a time and solve it on a separate piece of paper.
3. As teams complete one of these problems, they may be given another.

Your Comments for pages 2 A and 2 B:

_____

_____

_____

_____

1. FOR EARLY FINISHERS.

Mary has a favorite number. If you multiply it by 2 and add 4, you get 30. What is Mary's favorite number?

Lesson 2 A

2. FOR EARLY FINISHERS.

Terri has twice as much money as Walt. Together they have $1.80. How much money does Terri have?

Lesson 2 A

3. FOR EARLY FINISHERS.

The sum of two students' ages is 27 and the difference of their ages is 5 years. How old are they?

1. FOR EARLY FINISHERS.

Kim buys pencils and erasers. Pencils cost 12¢ and erasers cost 7¢. She bought twice as many pencils as erasers and spent 93¢. How many pencils did she buy?

Lesson 2 B

---

2. FOR EARLY FINISHERS.

The total weight of 3 football players is 705 pounds. Bill weighs 10 pounds more than Charly and Charly weighs 10 pounds more than Hank. How much does Hank weigh?

Lesson 2 B

---

3. FOR EARLY FINISHERS.

Kathy has 5 blocks. Some weigh 2 kilograms and some weigh 3 kilograms. The blocks weigh 14 kilograms altogether.

How many of her blocks weigh 2 kilograms? _____
How many weigh 3 kilograms? _____

LESSON 3 USING GUESSES AND DIAGRAMS TO SOLVE PROBLEMS

Purpose: Given a problem, the student will use guessing and diagraming to solve it.

Overview: In Lesson 3, we want the students to do things such as drawing pictures, connecting points, and drawing lines. The problems will usually require several guesses (diagrams) before the solution is found. We intend for the problems to increase the students' willingness to guess by drawing pictures. In some problems, the diagram is the solution, while in other problems the diagram leads to a solution.

Teaching Procedure:

Pages 9 and 10 may be worked by students, then discussed by the class.

Pages 11 and 12 may be worked by students at their own pace.

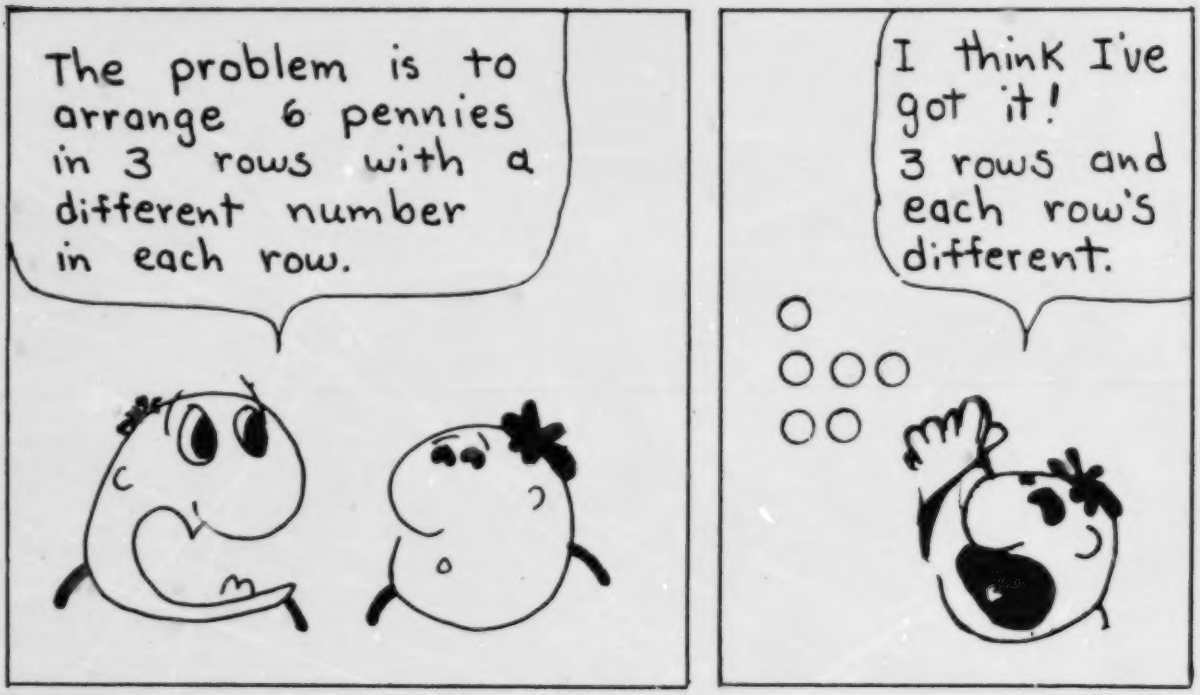
Pages 3 A and 3 B in the teacher's edition have problems which may be given to students who finish early.

Authors' Comments for page 9:

Read the cartoon and discuss the problem. Have the students do problem 1. Discuss the solutions to this problem and the need for a diagram. Have the students do problem 2, then discuss it.

Your Comments for page 9:

LESSON 3: USING GUESSES AND DIAGRAMS TO SOLVE PROBLEMS.


The problem is to arrange 6 pennies in 3 rows with a different number in each row.
I think I've got it! 3 rows and each row's different.

THESE PROBLEMS HAVE ANSWERS THAT ARE DIAGRAMS.

Problem 1. Show how 15 pennies can be arranged in 5 rows with the same number in each row.

(Hint: You can check your answers by seeing if you have used 15 pennies, if they are in 5 rows, and that there are the same number in each row.)

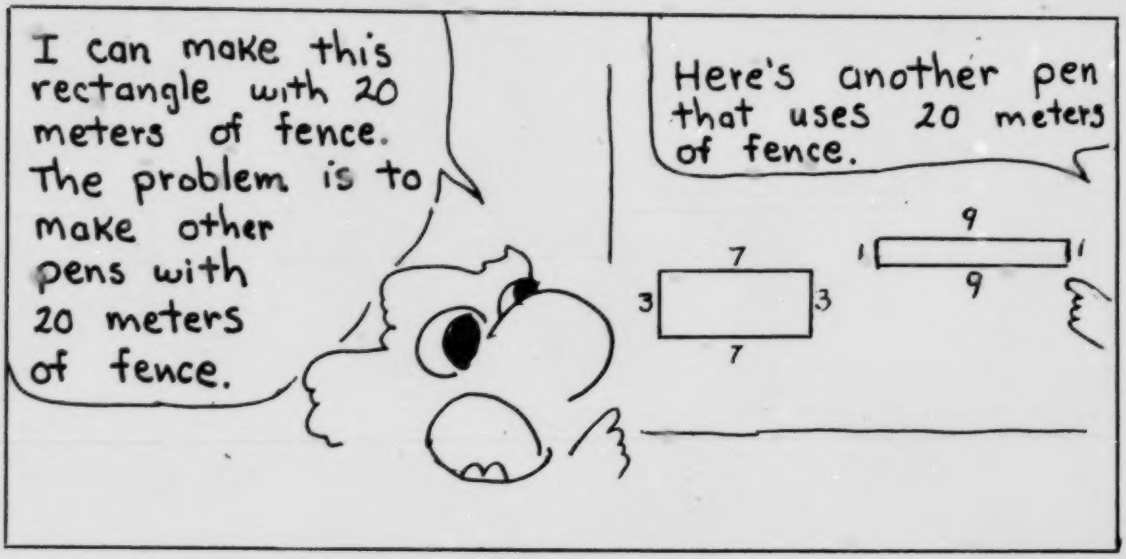
Problem 2. Show how 15 pennies can be arranged in 5 rows with a different number in each row.

(Hint: Check your diagram. Does it show pennies? Are they in 5 rows? Does each row have a different number of pennies?)

Authors' Comments for page 10:

Read the cartoon and discuss the setting. Discuss the idea that both rectangles in the cartoon use 20 meters of fence. Have the students do problem 1. The 3 rectangles in the solution to problem 1 are different from the 2 given in the cartoon. Discuss the rectangles which are solutions to problem 1. Have the students do problem 2 and discuss it.

Your Comments for page 10:



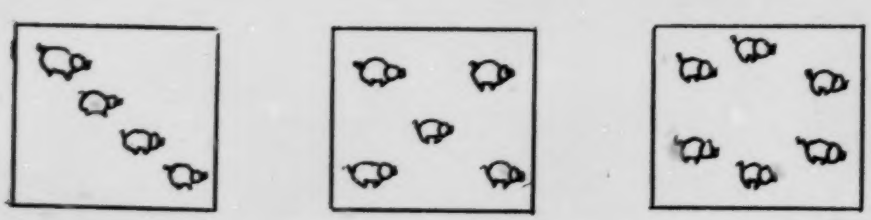
SOLVE THESE PROBLEMS WITH DIAGRAMS.

Problem 1. Draw 3 rectangular pens that use 20 meters of fence.

(Hint: Are your pens shaped in rectangles? Are they 20 meters around?)

Problem 2. Draw 3 straight lines in each picture so that each pig is in a separate pen.

(Hint: Did you draw 3 lines in each picture? Is each pig in a separate pen?)



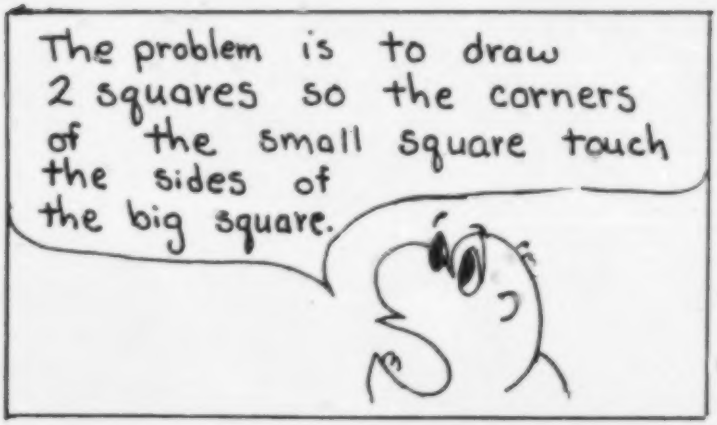
LESSON 3

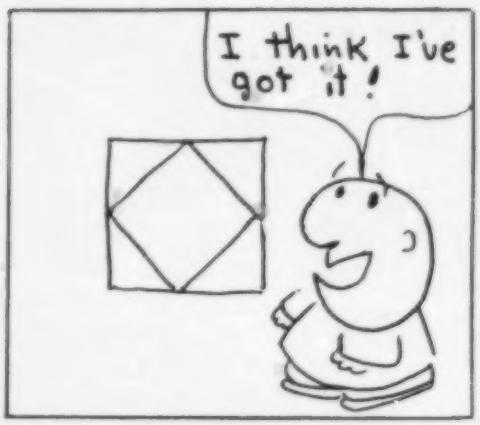
Page 11

Authors' Comments for page 11:

Students may work in teams on page 11. Little teacher direction is needed for this page. We suggest that the students discuss their answers with each other before going on to the next page.

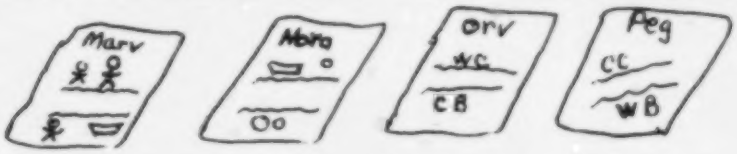
Your Comments for page 11:





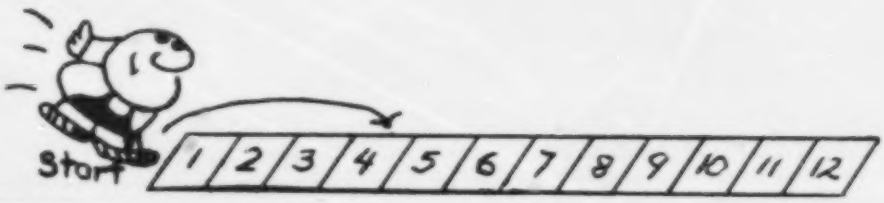
USE A DIAGRAM OR PICTURE TO SHOW THESE SITUATIONS.

1. A woman and child are on one side of a river and a child and boat are on the other side. Whose diagrams are correct?



2. Draw a circle, triangle, and square so the square is inside the circle and the triangle is inside the square.

3. Vince jumps forward 4 spaces, back 2, and forward 4. Use arrows to show his jumps.



LESSON 3

Page 12

Authors' Comments for page 12:

Students may work at their own pace on this page. Again we encourage them to discuss their answers with each other.

Your Comments on page 12:

SOLVE THESE PROBLEMS.

1. Draw 3 straight lines to cut the pizza into 7 pieces.

   Hint: The pieces don't have to be the same size.

2. A flag has four stripes going across--one blue, one green, one white, and one gold. Green is on top, white and gold are not together, and white is between green and blue. What color is the bottom stripe?

3. What is the largest number of crossing points you can get when you draw 4 straight lines?
   (Hint: The answer is larger than 4.)

LESSON 3

Pages 3 A and 3 B

OPTIONAL ACTIVITIES

Authors' Comments for pages 3 A and 3 B:

In our try-out experiences, we've found that lessons written for several age levels may be too long for some levels and too short for other levels. We have prepared these lessons so all students can complete the basic lesson in one period. We have provided optional activities for the early finishing groups. The optional activities for this lesson are labeled Lesson 3 A and Lesson 3 B. There are two copies of each of these pages. On each page are several problems. The problems on page 3 A (printed on blue) should be given to average and below average students. The problems on page 3 (printed on yellow) are a little more difficult and should be given to above average students.

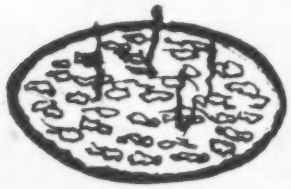
1. Cut the problems apart on the dotted lines.
2. Early finishers get one problem at a time and solve it on a separate piece of paper.
3. As teams complete one of these problems, they may be given another.

Your Comments for pages 3 A and 3 B:

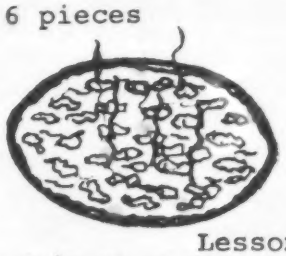
1. FOR EARLY FINISHERS.

Draw 3 straight lines to cut each pizza in these pieces.

4 pieces



6 pieces



Lesson 3 A

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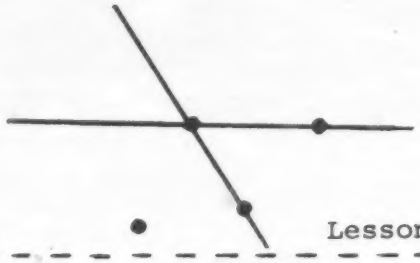
2. FOR EARLY FINISHERS.

Draw all the lines which connect 2 dots. Two lines are already drawn. How many lines should be drawn in all? _____



Lesson 3 A

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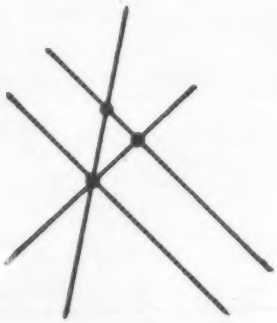
3. FOR EARLY FINISHERS.

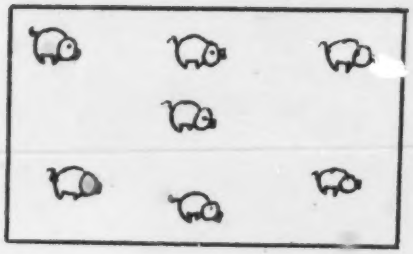


This shows 4 lines crossing at 3 points.

Draw 4 lines crossing at 5 points.

1. FOR EARLY FINISHERS.

Draw 3 straight lines so that each pig is in a separate pen.



Lesson 3 B

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2. FOR EARLY FINISHERS.

A well is 9 meters deep. A frog climbs 3 meters during the day, but slides back 1 meter during the night. The frog starts at the bottom of the well. On which day does it get to the top?

Lesson 3 B

---

3. FOR EARLY FINISHERS.

You have 2 circles and a square. A circle is inside the square and the square is inside a circle. Draw a diagram to show how they are arranged.

LESSON 4 USING ESTIMATION TO SOLVE MEASUREMENT PROBLEMS

Purpose: Given a problem and some information, the student will use the information to estimate a solution.

Overview: In Lesson 4, we want students to solve problems in which they use some given information to estimate solutions. Because of feedback from earlier versions of this lesson, we have:

1. Written the lesson to focus on a given piece of information and the use of that information to make estimates.

2. Written the lesson so that students will not be required to use personal or physical measurements to make estimates.

Because of the nature of estimation, students will likely disagree on some of the solutions. For each problem, teachers should accept any solutions in a reasonable interval.

Teaching Procedure:

Pages 13 and 14 are teacher-directed.

Pages 15, 16, and 17 may be worked by the students at their own pace. With less able classes the teacher may choose to provide direction on these pages as well.

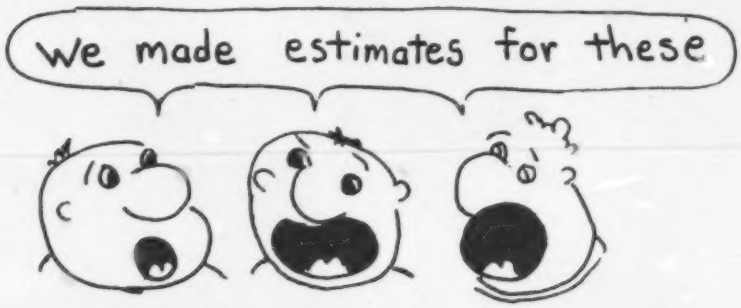
Pages 4 A and 4 B in the teacher's edition have problems which may be given to students who finish early.

Authors' Comments for page 13:

Have students circle the best estimate for each problem. Discuss their estimates and have the students describe how they decided on their estimate. After the discussion of a problem, students may wish to change their estimate. This is fine.

Your Comments for page 13:

LESSON 4: USING ESTIMATION TO SOLVE MEASUREMENT PROBLEMS.

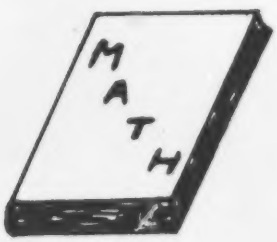


Circle the "best" estimate. Be prepared to discuss how you decided.

Problem 1.

How high is the flag pole?

Jill, "About 40 feet."
John, "About 10 feet."
Matt, "About 20 feet."



Problem 2.

How many pages are in the math book?

Jill, "300 to 400 pages."
John, "100 to 200 pages."
Matt, "600 to 700 pages."

Problem 3.

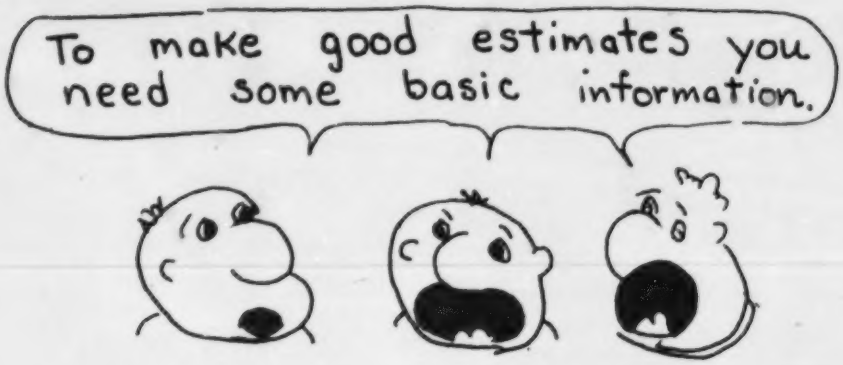
Two nickels balance a new piece of chalk. How much does a new piece of chalk weigh?

Jill, "About 100 grams."
John, "About 10 grams."
Matt, "About 1 gram."

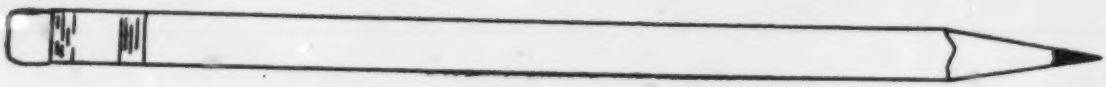
Authors' Comments for page 14:

Read the cartoon and briefly discuss the importance of using information to help make better estimates. Have students work the pencil problems on page 14, then discuss their estimates. After this discussion, students may want to change their estimates. This is fine. Repeat the process for problems 1-3.

Your Comments for page 14:

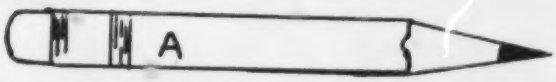


You know:

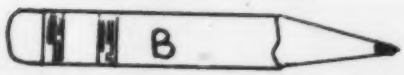


A new pencil is 16 centimeters long.

Estimate the length of these pencils.



_____cm



_____cm

_____cm

Look at the door of your room. It is about 2 meters high. Estimate these.

1. How high is the ceiling of your classroom? _____ meters
2. How many meters long is the chalkboard? _____ meters
3. How many meters wide is your classroom? _____ meters

Authors' Comments for page 15:

Students may work at their own pace on this page. We encourage them to discuss their answers with each other.

Your Comments for page 15:

5 grams

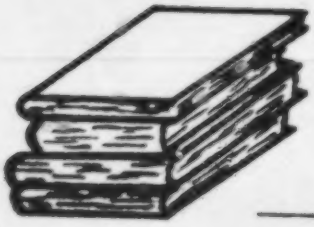
250 grams

1 kilogram
(1,000 grams)

1. Estimate the weight of these. Be prepared to discuss how you made your estimate.

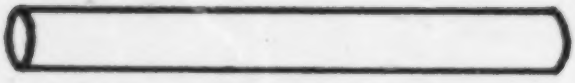
_____ grams

_____ grams



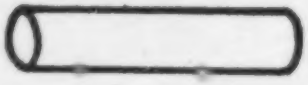
_____ kilograms

_____ grams



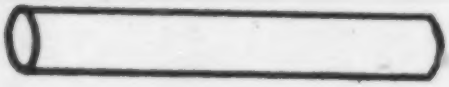
A new piece of chalk weighs about 10 grams.

2. Estimate the weight of these pieces:

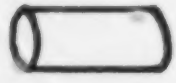


_____ grams

_____ grams



_____ grams



_____ grams

LESSON 4

Page 16

Authors' Comments for page 16:

Students may work at their own pace on this page. We encourage them to discuss their answers with each other.

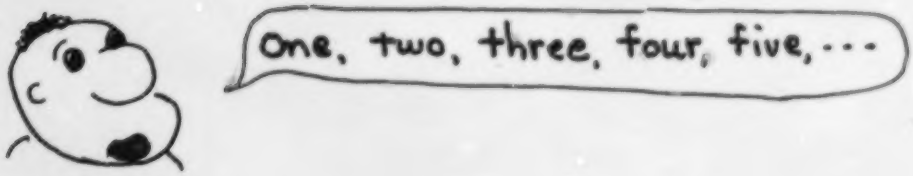
Your Comments for page 16:

Use the saying to estimate the weight of these:

_____ 1. A quart of water (Remember 2 pints make a quart).

_____ 2. A gallon of water (Remember 4 quarts make a gallon).

_____ 3. The weight of gasoline in a full 25 gallon gas tank (Gasoline is a little lighter than water).



Find out how far you can count by ones in a minute. Use your information to estimate how long it would take you:

1. To count by ones to 500. _____ minutes

2. To count by ones to 1,000. _____ minutes

3. About how far could you count by ones in one hour? _____

Authors' Comments for page 17:

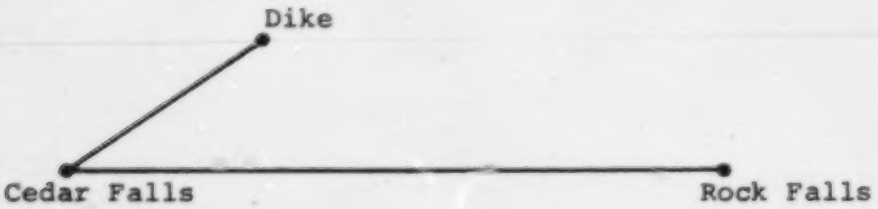
Students may work at their own pace on this page. We encourage them to discuss their answers with each other.

Your Comments for page 17:

USE ESTIMATION TO SOLVE THESE PROBLEMS.

There are no correct answers, but some estimates are better than others.

1.



It is 20 miles from Cedar Falls to Dike. Estimate how far it is from Cedar Falls to Rock Falls. _____ miles

2.

The seat of the chair is 45 centimeters from the floor. Estimate the height of the chair. _____ centimeters

3.

The book mark is on page 100. Estimate the number of pages in the book. _____ pages

4.

It took the snail 12 minutes to crawl from A to C. Estimate how long it took the snail to crawl from A to B.

LESSON 4

Pages 4 A and 4 B

OPTIONAL ACTIVITIES

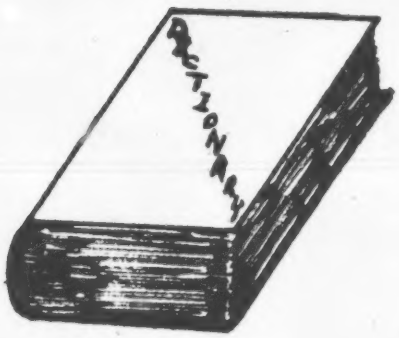
Authors' Comments for pages 4 A and 4 B:

In our try-out experiences, we've found that lessons written for several age levels may be too long for some levels and too short for other levels. We have prepared these lessons so all students can complete the basic lesson in one period. We have provided optional activities for the early finishing groups. The optional activities for this lesson are labeled Lesson 4 A and Lesson 4 B. There are two copies of each of these pages. On each page are several problems. The problems on page 4 A (printed on blue) should be given to average and below average students. The problems on page 4 B (printed on yellow) are a little more difficult and should be given to above average students.

1. Cut the problems apart on the dotted lines.
2. Early finishers get one problem at a time and solve it on a separate piece of paper.
3. As teams complete one of these problems, they may be given another.

Your Comments for pages 4 A and 4 B:

1. FOR EARLY FINISHERS.



This dictionary has 500 pages. Estimate the page where you would find these words:

Vitamin
Crystal
Mosquito
Secret

Lesson 4 A

2. FOR EARLY FINISHERS.

It takes 7 gallons of gasoline for the truck to travel from town A to town B. Estimate how much gasoline it takes the truck to travel from A to C.

Lesson 4 A

3. FOR EARLY FINISHERS.

A one dollar bill weighs about 1 gram.

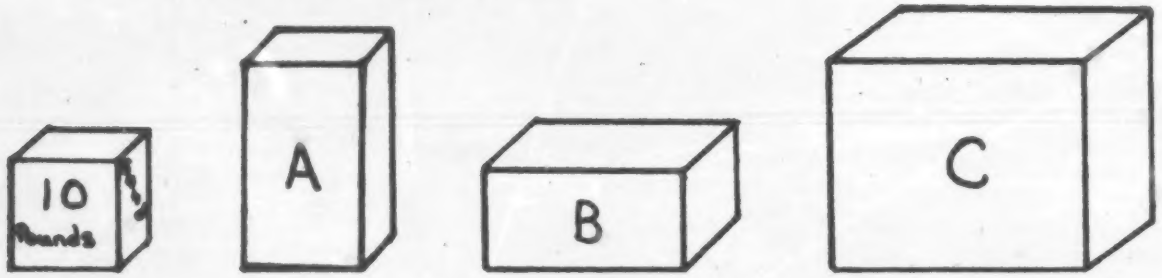
A. What is the weight of $100 in one dollar bills?

B. If a stack of one dollar bills weighs 550 grams, about how much money is in the stack?

C. How much money is in a stack of $10 bills that weighs 50 grams? (Ten dollar bills weigh the same as dollar bills.)

Lesson 4 A

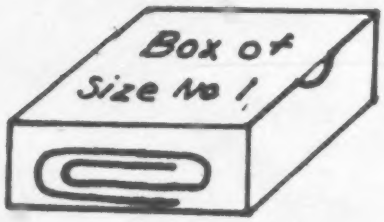
1. FOR EARLY FINISHERS.



The small box contains 10 pounds of sand. Estimate the weight of the sand necessary to fill the other boxes.

Lesson 4 B

2. FOR EARLY FINISHERS.



A paper clip of size 1 weighs about 1 gram and is about 3 centimeters long. You have a box of size 1 paper clips that weighs about 100 grams. About how long a string of paper clips could you make from the clips in the box?

LESSON 5 USING GUESSES TO SOLVE PROBLEMS

Purpose: Given a problem of the type encountered in the first four lessons, the student will use guesses or estimates to solve it.

Overview: Lesson 5 serves as a bridge between the first four lessons and the problem card deck. The problems the student solves in this lesson are of the various types they encountered in Lessons 1-4. They are presented in the format used in the card deck. The students will not do all the problems in this lesson, but will select certain ones to work on.

Teaching Procedure:

Page 18 is teacher-directed.

Pages 19-24 are worked by students without teacher direction. Each student will work on two of these pages.

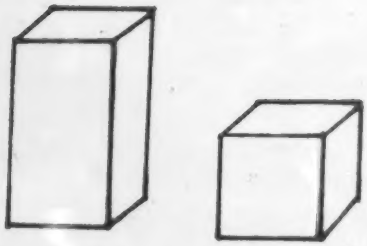
Authors' Comments for page 18:

Explain that in this lesson they will solve problems similar to some they have solved before. First everyone will solve the two problems on page 18. Then they may select either the Blue, the Yellow, or the Red problems. The Blue problems are about as difficult as the ones on page 18. The Yellow problems are a little more difficult and the Red problems are the most difficult. After the students have finished the problems of their choice, they should then select a problem from the problem deck to work on for the remainder of the period.

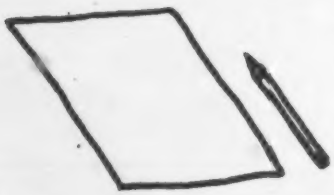
Your Comments for page 18:

LESSON 5: USING GUESSES TO SOLVE PROBLEMS.

EVERYONE SOLVE THESE TWO PROBLEMS.



The small boxes are 20 cm high and the large boxes are 40 cm high. A stack of boxes is 100 cm high. How many of each size are in the stack?



The paper is 28 cm long. Guess the length of the pencil.

NOW CHOOSE THE BLUE, YELLOW, OR RED PROBLEMS.

LESSON 5

Pages 19-20

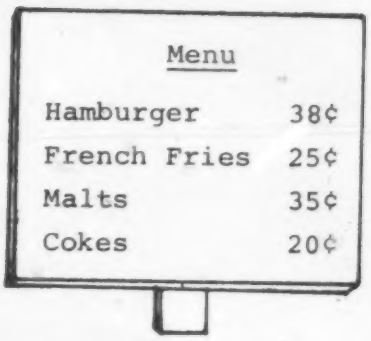
Authors' Comments for pages 19 and 20:

These pages are printed on Blue paper and the problems are similar to those on the Blue cards in the problem deck. These are the easiest of the problems on the three colored pages, but they are by no means trivial.

When the students finish the two pages of problems on the Blue sheets, they should select problems from the problem solving deck. This move to the problem deck eliminates the pile up that occurs when all of the students in the class go to the deck at the same time.

Your Comments for pages 19 and 20:

42BC6



43BD6

Fence posts are 3 meters apart. Use a diagram to show how many fence posts you use to put up 30 meters of fence in a straight line. How many fence posts are used?

44DMD

Bonnie is 5 feet tall. How tall is her sign?

45BV6

Solve the puzzle. I am twice as old as my sister. Together our ages are 15. How old is my sister?

LESSON 5

Pages 21-22

Authors' Comments for pages 21 and 22:

These two pages are printed on Yellow paper and the problems are similar to those on the Yellow cards in the problem deck. These are more difficult than the Blue problems, but not as difficult as the Red problems.

When the students finish the two pages of problems on the Yellow sheets, they should select problems from the problem solving deck. This move to the problem deck eliminates the pile up that occurs when all of the students in the class go to the deck at the same time.

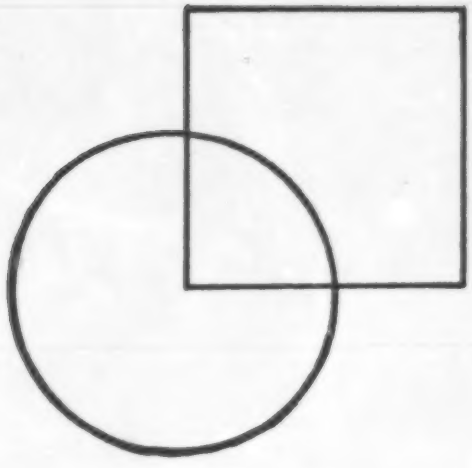
Your Comments for pages 21 and 22:

40YV6

A bowl has 24 pieces of fruit. Some are oranges and some are grapefruit. It has twice as many oranges as grapefruit. How many oranges are in the bowl?

41YD6

Draw 8 dots so that 5 dots are inside the circle and 5 dots are inside the square.

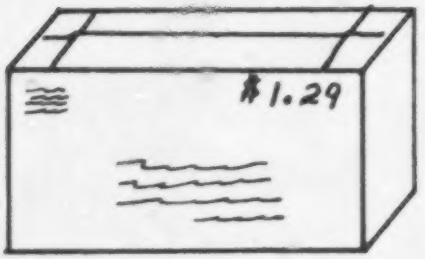


42YM6

Estimate the height of the building. The basket is 10 feet above the ground.

43YC6

Which of these stamps would you use to mail the box?



LESSON 5

Pages 23-24

Authors' Comments for pages 23 and 24:

These two pages are printed on Red paper and the problems are similar to those on the Red cards in the problem deck. These are the most difficult of the problems on the three colored pages. When the students finish the two pages of problems on the Red sheets, they should select problems from the problem solving deck. This move to the problem deck eliminates the pile up that occurs when all of the students in the class go to the deck at the same time.

Your Comments for pages 23 and 24:

40RV6

Lonny has 2 bats and 1 baseball.
They cost $11 altogether.

Melanie has 1 bat and 2 baseballs.
They cost $7 altogether.

How much does 1 bat cost? _____

How much does 1 baseball cost? _____

41RM6

This page is 28 cm long. Estimate its width.

42RD6

I am thinking of a number. If I multiply the number times itself, I get a product of 625. What number am I thinking of?

43RD6

A well is 30 meters deep. A frog climbs up 7 meters during the day. but slips back 2 meters during the night. If the frog starts at the bottom of the well, on which day does he get to the top?